MATHEMATICAL APPENDIX
A Two-Part Feed-In-Tariff for Intermittent Electricity Generation

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The derivation of optimal FIT and CAT gives rise to a number of predictions, which are summarized in this appendix. Equation (12) demonstrates that a two-part tariff is often feasible but not always feasible. The electric utility can use either a FIT or a CAT or both. The following theorems identify properties that follow from (12).

Theorem 1 (Existence). *For a feasible combination of FIT and CAT to exist, it must hold that $f_n \leq z_n$. Because $f_n > 0$, a feasible two-part tariff does not exist if $z_n < 0$.*

This first theorem has an important implication. Because z_n must be non-negative, it must hold that $p_0T/s_0 > \gamma v_n/u_n - 1$. This means that the two-part tariff is more likely to be feasible when the v_n is small or negative, or when the utilization rate u_n is large. There is an obvious corollary. When the ratio v_n is negative, a feasible two-part tariff always exists because the expression $\gamma v_n/u_n - 1$ is negative. The second corollary follows from the fact that p_0 and s_0 are positive, and thus $p_0T/s_0 > 0$. Therefore, a feasible two-part tariff also exists always when v_n is positive but sufficiently small: $v_n < u_n/\gamma$.

The next two theorems explore the constraints on offering single-part tariffs.

Theorem 2 (FIT without CAT). *If the utility does not offer a CAT, then it must offer a differentiated FIT p_n such that $f_n/u_nT \leq p_n \leq z_n/u_nT$.*

Theorem 3 (CAT without FIT). *If the utility does not offer a FIT, then it must offer a differentiated CAT such that $f_n \leq s_n \leq z_n$.*

If the utility offers a two-part tariff, there is an equivalency relationship between the two components, defined in the next theorem.

Theorem 4 (Equivalence). *If the utility offers both FIT and CAT, any incentive pair (p_n, s_n) and (p'_n, s'_n) that satisfies $(s_n - s'_n)/(p'_n - p_n) = u_nT$ is equivalent.*

The next two theorems explore the possibility that the utility offers a two-part tariff that is uniform across IPPs in one element and differentiated in the other element.

Theorem 5 (Uniform FIT). *If the utility pays a uniform FIT $\forall n : p_n = \tilde{p}$, it must offer a differentiated CAT s_n so that $f_n - \tilde{p}u_nT \leq s_n \leq z_n - \tilde{p}u_nT$.*

Theorem 6 (Uniform CAT). *If the utility pays a uniform CAT $\forall n : s_n = \tilde{s}$, it must offer a differentiated FIT p_n such that $f_n - \tilde{s} \leq p_nu_nT \leq z_n - \tilde{s}$.*

A useful comparison concerns the conventional scenario of a uniform FIT without a second pricing instrument

Theorem 7 (Conventional FIT). *A uniform FIT $\forall n : p_n = \tilde{p}$ without CAT $\forall n : s_n = 0$ is welfare-reducing because it rejects projects n for which $\tilde{p} < f_n/(u_nT) < \tilde{p} + s_n/(u_nT)$ when s_n would have been positive, and it accepts projects n for which $\tilde{p} + s_n/(u_nT) < z_n/(u_nT) < \tilde{p}$ when s_n would have been negative.*

In other words, a uniform FIT without CAT falsely rejects projects when $\tilde{p}$ is not crediting new projects for their reduction in back-up capacity, and it falsely accepts projects when $\tilde{p}$ is not charging new projects for an increase in back-up capacity. Essentially, a uniform FIT without CAT generates type-I and type-II errors.

The final theorem looks at a variation of theorem 5 where the utility offers a uniform FIT, but with a differentiated CAT. This option has some desirable characteristics because it attributes the heterogeneity across firms to one pricing element only. It fixes the problem encountered in theorem 7.

Theorem 8 (Optimal uniform FIT with differentiated CAT). *Among the feasible two-part tariffs is one that offers a uniform FIT defined by $\forall n : p_n = \tilde{p} = p_0$ and a heterogeneous CAT defined by $s_n \leq s_0(u_n - \gamma v_n)$. The CAT is positive when $u_n > \gamma v_n$ and is negative when $u_n < \gamma v_n$. When v_n is negative, the CAT is guaranteed to be positive.*

Using (7) and (11), the sufficiency condition for the CAT to be positive is

$$\frac{u_nK_n}{\gamma\sqrt{\Sigma_0^{(n-1)}}} > \sqrt{1 + \frac{\Delta^{(n)}}{\Sigma_0^{(n-1)}}} - 1 \qquad (18)$$

When $\Delta^{(n)} < 0$, the expression on the right-hand side of (18) is negative, and thus the inequality always holds. But even for small positive $\Delta^{(n)}$, the inequality will hold.